Switching of electromagnetic waves by two-layered periodic dielectric structures

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The propagation of electromagnetic waves in two-layered periodic dielectric structures is investigated. The systematic dependence of the reflection coefficient on the parameters characterizing the structure is studied in detail. Using results of this exact analysis we investigate the influence of variation of the structure parameters on the reflection and transmission coefficients. As an example, we consider the changes in the structure parameters under an elastic stress. We show that under practically realizable conditions the reflection and transmission of the electromagnetic wave can be changed by as much as 80–90 % by creating a constant elastic stress inside the structure. [S1063-651X(99)01210-6]

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I. INTRODUCTION

The propagation of waves through media with one-dimensional periodicity has long been a topic of interest in various areas of physics and technology, beginning with the well-known paper of Kronig and Penney [1]. The literature on the subject includes several monographs and reviews [2–7] which are mostly devoted to the development of effective numerical and approximate analytical methods. They include the Floquet-Bloch method [8], the matrix method [5,9,10], the theory of Kogelnik coupled waves [11], and some modifications of theory of coupled waves [7,12,13]. Recent studies in this area have led to important new analytic results for one-dimensional periodic structures with an arbitrary periodic shape for profile of the dielectric permittivity. Results have been obtained for the transmission coefficient [14–16], field distribution [15,16], and group velocity [15,16] for electromagnetic waves propagating normally to the axis of periodicity through periodic structure. Some of these results can be extended to general one-dimensional inhomogeneous media [16].

From a mathematical point of view, layered periodic structures have a special interest because this is one of the few cases [17] where it is possible to find exact analytic solutions. However, even for the simplest case of a two-layered periodic structure, previous solutions in terms of Floquet-Bloch waves were very complicated and cast in a form involving several parameters whose physical meaning was not clear. As a result in existing expressions for the reflection and transmission coefficients for waves incident on two-layered periodic structures [5,12] it is difficult to follow analytically the dependence of these coefficients on the structure parameters, such as refractive indices of the basic layers and their widths.

Despite the fact that two-layered periodic structures have been extensively used in different physical applications for more than 30 years, some new possibilities for their practical use follow from a more thorough theoretical analysis. In recent papers [18,19] it was suggested that two-layered periodic dielectric structures could be used as a dielectric omnidirectional reflector. Such an application does indeed follow directly from a thorough analysis of forbidden and allowed regions of optical frequencies for the structure.

In this paper we develop an improved analytic formulation for the propagation of two Floquet-Bloch waves inside a two-layered periodic structure for the general case of plane waves incident at an arbitrary angle to the axis of periodicity. We extract exact analytic expressions for the reflection and transmission coefficients for the case of normal angle of incidence in a more simple and physically understandable form. Using a more convenient set of parameters we obtained exact expressions for allowed and forbidden regions of frequencies for the structure. As a result we find extremely high sensitivity of the reflection and transmission of the electromagnetic wave to the structure parameters. For example, in some cases it is possible to change them by up to 80% by varying the structure parameters by only 0.1%. As the simplest possibility for the creation of such variation, we consider in detail the result of applying a constant elastic stress inside the structure. We have identified suitable combinations of materials for the construction of practical optical switching devices based on this effect.

In the following section, the method of exact analytic solution is developed in terms of Floquet-Bloch waves propagating inside the structure. These results are then applied in Sec. III to calculate the reflection and transmission coefficients for the case of normal incidence for the incoming waves. In Sec. IV we investigate the influence of a constant elastic stress on the structure parameters, and as a result on the reflection and transmission of electromagnetic waves propagating through the structure. Finally, the conclusions are summarized in Sec. V.

II. EXACT ANALYTIC SOLUTION IN TERMS OF FLOQUET-BLOCH WAVES

The parameters specifying the problem are as shown in Fig. 1. We assume a two-layered periodic transparent (without absorption) nonmagnetic ($\mu=1$) medium with refractive indices n_1 and n_2 and thicknesses d_1 and d_2 of the layers such that $d=d_1+d_2$ is the period of the function $n(z)$ $[n^2(z)=\varepsilon(z)]$, and N is the number of the periods of the structure. Monochromatic plane waves with angular frequency ω and wave vector in vacuum $\mathbf{k}_0$ are assumed to propagate inside the structure in the xz plane according to Maxwell's equations

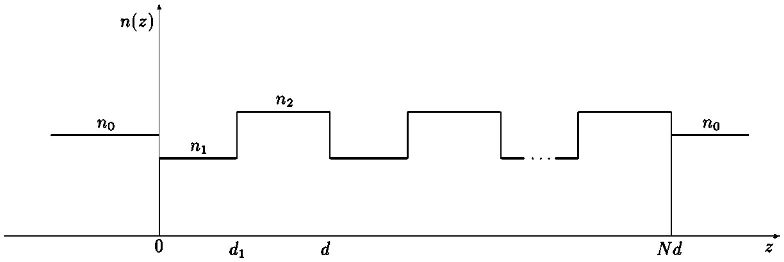

FIG. 1. Two-layered periodic dielectric structure.

$$\nabla \times \mathbf{E} = \frac{i\omega}{c}\mathbf{H}, \quad \nabla \times \mathbf{H} = -i\varepsilon(z)\frac{\omega}{c}\mathbf{E}. \tag{1}$$

The solutions are then independent of the coordinate y. However, the total electric field $E(x,z,t)$ inside the structure can have two independent polarizations. In the first case, $\mathbf{E}$ is perpendicular to the plane of wave propagation, i.e., it is directed along the y axis, and magnetic field $\mathbf{H}$ is in the xz plane of propagation. In the second case $\mathbf{H}$ is directed along the y axis and $\mathbf{E}$ lies in the xz plane of propagation.

In this paper we consider only the first case of polarization. The solutions to Eq. (1) for the total electric field inside the structure can then be expressed as the linear superposition

$$\mathbf{E}(x,z,t) = E(z)[D_1 \exp(i\theta x) + D_2 \exp(-i\theta x)] \times \exp(-i\omega t)\hat{\mathbf{e}}_y, \tag{2}$$

where $\hat{\mathbf{e}}_y$ is a unit vector along the y axis, $\theta = k_0 n_1 \sin\alpha_1 = k_0 n_2 \sin\alpha_2$ are projections of the wave vector $\mathbf{k}$ onto the x axis in the layers with refractive indexes n_1, n_2, respectively, and α_1 and α_2 are the angles between the direction of wave propagation and the z axis in these layers. The function $E(z)$ is then determined by the equation

$$\frac{d^2E(z)}{dz^2} + [k_0^2\varepsilon(z) - \theta^2]E(z) = 0, \tag{3}$$

where $\varepsilon(z)$ is a piecewise constant periodic function. Thus Eq. (3) has the form of the Hill equation and according to Floquet theory [2,8] we can represent its general solution as a superposition of two traveling Floquet-Bloch waves in the form

$$E(z) = C_1 E_1(z) + C_2 E_2(z), \tag{4}$$

where

$$E_{1,2}(z) = F_{1,2}(z)\exp\left(i\frac{\xi_{1,2}}{d}z\right), \quad F_{1,2}(z) = F_{1,2}(z+d). \tag{5}$$

The quantities $\xi_{1,2}$ are the so-called characteristic Floquet indices, related by $\xi_2 = -\xi_1$ [20]. To find exact analytic expressions for the Floquet-Bloch waves $E_1(z)$ and $E_2(z)$, we represent them inside the n_1 layer of the first period as

$$E_{1,2}(z) = A_{1,2}\sin(\theta_1 z - \tfrac{1}{2}\theta_1 d_1 + \varphi_{1,2}), \tag{6}$$

where $\theta_1 = \sqrt{(k_0^2\varepsilon_1 - \theta^2)}$ is the z component of the wave vector $\mathbf{k}$ in the n_1 layer. Analogously, inside the n_2 layer of the same period,

$$E_{1,2}(z) = B_{1,2}\sin[\theta_1(z-d_1) - \tfrac{1}{2}\theta_2 d_2 + \psi_{1,2}], \tag{7}$$

where $\theta_2 = \sqrt{(k_0^2\varepsilon_2 - \theta^2)}$ is the z component of the wave vector $\mathbf{k}$ in the n_2 layer. By direct substitution, it is clear that these functions satisfy Eq. (3). The coefficients A_1 and A_2 in formulas (7) can be set equal to unity (they play the role of normalization constants for the Floquet-Bloch waves). The phases $\varphi_{1,2}$ and $\psi_{1,2}$ are in general complex. Their introduction represents the major point of departure from the usual representation of Floquet-Bloch waves inside each layer as a superposition of two exponential functions [3,5,9,12].

To find the parameters φ_1, ψ_1, and B_1 which define Floquet-Bloch wave $E_1(z)$ in the first period, we use the conditions of continuity for the function $E_1(z)$ and its derivative (corresponding to the continuity of the magnetic field) at the points $z = d_1$ and $z = d_1 + d_2 \equiv d$ and then use the Floquet theorem in the form [2,8,20] $E_1(z) = \exp(i\xi_1)E_1(z-d)$, which means that in systems with periodic coefficients, the solutions differ by a factor $\exp(i\xi)$ upon shifting by the period d. As a result we have the system

$$\begin{aligned}
&\sin(\tfrac{1}{2}\theta_1 d_1 + \varphi_1) = B_1 \sin(-\tfrac{1}{2}\theta_2 d_2 + \psi_1),\\
&\theta_1 \cos(\tfrac{1}{2}\theta_1 d_1 + \varphi_1) = \theta_2 B_1 \cos(-\tfrac{1}{2}\theta_2 d_2 + \psi_1),\\
&\exp(i\xi_1)\sin(-\tfrac{1}{2}\theta_1 d_1 + \varphi_1) = B_1 \sin(\tfrac{1}{2}\theta_2 d_2 + \psi_1),\\
&\theta_1 \exp(i\xi_1)\cos(-\tfrac{1}{2}\theta_1 d_1 + \varphi_1) = \theta_2 B_1 \cos(\tfrac{1}{2}\theta_2 d_2 + \psi_1).
\end{aligned} \tag{8}$$

The second Floquet-Bloch wave $E_2(z)$ is determined by an analogous procedure with the parameters φ_1, ψ_1, ξ_1, B_1 replaced by φ_2, ψ_2, ξ_2, B_2. Thus, system (8) describes both Floquet-Bloch waves $E_1(z)$ and $E_2(z)$. To emphasize this point, we may drop the subscripts on the unknown values φ, ψ, ξ, B, and make the following transformations. Divide the first equation into the second and the third into the fourth to obtain the system of two equations and two unknown parameters φ, ψ:

$$\begin{aligned}
&\tan(\varphi + \tfrac{1}{2}\theta_1 d_1) = \frac{\theta_1}{\theta_2}\tan(\psi - \tfrac{1}{2}\theta_2 d_2),\\
&\tan(\varphi - \tfrac{1}{2}\theta_1 d_1) = \frac{\theta_1}{\theta_2}\tan(\psi + \tfrac{1}{2}\theta_2 d_2).
\end{aligned} \tag{9}$$

Before finding the solution of this system, we define the following more convenient parameters:

$$a=\frac{\theta_2-\theta_1}{\theta_2+\theta_1}=\frac{\sqrt{k_0^2\varepsilon_2-\theta^2}-\sqrt{k_0^2\varepsilon_1-\theta^2}}{\sqrt{k_0^2\varepsilon_2-\theta^2}+\sqrt{k_0^2\varepsilon_1-\theta^2}},$$

$$\Omega=\theta_2 d_2+\theta_1 d_1,$$

$$\Delta=\theta_2 d_2-\theta_1 d_1, \tag{10}$$

$$C=\frac{\sin(\frac{1}{2}\Omega)-a\sin(\frac{1}{2}\Delta)}{\sin(\frac{1}{2}\Omega)+a\sin(\frac{1}{2}\Delta)}.$$

Then we solve system (9) and express the final solutions for parameters of Floquet-Bloch waves $E_1(z)$ and $E_2(z)$ in terms of a, Ω, Δ. We have if $C>0$

$$\varphi_{1,2}=\pm\tfrac{1}{2}\arccos\left[\frac{a^{-1}\sin\Omega+a\sin\Delta}{2\sin(\theta_2 d_2)}\right],$$

$$\psi_{1,2}=\mp\tfrac{1}{2}\arccos\left[\frac{-a^{-1}\sin\Omega+a\sin\Delta}{2\sin(\theta_1 d_1)}\right]. \tag{11}$$

If $C<0$, our parameters are defined by

$$\varphi_{1,2}=\pm\tfrac{1}{2}\arccos\left[\frac{a^{-1}\sin\Omega+a\sin\Delta}{2\sin(\theta_2 d_2)}\right],$$

$$\psi_{1,2}=\pm\tfrac{1}{2}\arccos\left[\frac{-a^{-1}\sin\Omega+a\sin\Delta}{2\sin(\theta_1 d_1)}\right]. \tag{12}$$

We now return to a discussion of the characteristic Floquet indices $\xi_{1,2}$ ($\xi_2=-\xi_1\equiv\xi$). The well-known dispersion equation for the $\xi_{1,2}$ [3,5,6,9,12] follows from system (8). In terms of a, Ω, and Δ, the dispersion equation is

$$\cos\xi=\frac{1}{1-a^2}\cos\Omega-\frac{a^2}{1-a^2}\cos\Delta. \tag{13}$$

Its analysis allows one to establish the basic properties of Floquet-Bloch waves [3,5,12]. There are two physically different regions of parameters for our structure. In the first, the $\xi_{1,2}$ are real [$|\cos(\xi)|\leqslant 1$], and the waves $E_{1,2}(z)$ propagate without attenuation. Such regions are called allowed regions. In the second, the $\xi_{1,2}$ are complex [$|\cos(\xi)|\geqslant 1$], and the $E_1(z)$ direct Floquet-Bloch wave is exponentially damped, even in the absence of real absorption. Such regions are called forbidden regions. Physically, the accumulated Fresnel reflection from inhomogeneities of the dielectric permittivity $\varepsilon(z)$ with the periodicity of the structure results in an increase of the amplitude of the backward Floquet-Bloch wave at the expense of the forward wave, leading to an increase of the reflection coefficient. Using the condition $|\cos(\xi)|\geqslant 1$ we can easily find from Eq. (13) the widths of forbidden regions:

$$\Delta\Omega_{\text{odd}}=2\arccos[1-2a^2\cos^2(\tfrac{1}{2}\Delta)], \tag{14}$$

$$\Delta\Omega_{\text{even}}=2\arccos[1-2a^2\sin^2(\tfrac{1}{2}\Delta)], \tag{15}$$

where $\Delta\Omega_{\text{odd}}$ and $\Delta\Omega_{\text{even}}$ are the widths of odd [with the centers $\Omega=(2l+1)\pi, l=0,1,2,\ldots$] and even (with the centers $\Omega=2l\pi$) forbidden regions. Expressions (14) and (15) will play an extremely important role in the investigation of the elastic stress influence on the electromagnetic wave propagation in Sec. IV. Also we would like to emphasize that they are exact.

However, our aim now is to express the Floquet-Bloch waves $E_{1,2}(z)$ in a more transparent form directly in terms of the already known parameters $\varphi_{1,2}$ and $\psi_{1,2}$. This cannot be done from Eq. (13) as it stands because the sign of ξ_1 and ξ_2 remains ambiguous. In order to rectify this, we express ξ_1 and ξ_2 directly from the system (8) in terms of parameters φ_1,ψ_1 and φ_2,ψ_2, and then use Eqs. (11) and (12). The final result is

$$\xi_{1,2}=\pm\frac{a\sin[\frac{1}{2}(\Omega+\Delta)]}{|a\sin[\frac{1}{2}(\Omega+\Delta)]|}\times\arctan\left[\frac{i\sqrt{(\cos\Omega-a^2\cos\Delta)^2-(1-a^2)^2}}{\cos\Omega-a^2\cos\Delta}\right], \tag{16}$$

which is equivalent to Eq. (13) except that the sign of $\xi_{1,2}$ is now determined.

Thus, we have defined all the parameters (ξ_1, φ_1, ψ_1 and ξ_2, φ_2, ψ_2) which determine both Floquet-Bloch waves in the first period of our structure. The normalization coefficients B_1 and B_2 in the second layer of the first period can be expressed in terms of φ_1, ψ_1, and φ_2, ψ_2 from any equation of the system (8). Using Floquet's theorem for the two waves in the form $E_1(z)=\exp[i\xi_1(N-1)]E_1(z-Nd)$ and $E_2(z)=\exp[i\xi_2(N-1)]E_2(z-Nd)$, we can obtain final expressions for them in the Nth period of the structure as

$$E_1(z)=\sin\{\theta_1[z-(N-1)d]-\tfrac{1}{2}\theta_1 d_1+\varphi_1\}\times\exp[i\xi_1(N-1)],$$

$$E_2(z)=\sin\{\theta_1[z-(N-1)d]-\tfrac{1}{2}\theta_1 d_1+\varphi_2\}\exp[i\xi_2(N-1)] \tag{17}$$

in the layers with $n=n_1$, and

$$E_1(z)=B_1\sin\{\theta_2[z-d_1-(N-1)d]-\tfrac{1}{2}\theta_2 d_2+\psi_1\}\exp[i\xi_1(N-1)],$$

$$E_2(z)=B_2\sin\{\theta_2[z-d_1-(N-1)d]-\tfrac{1}{2}\theta_2 d_2+\psi_2\}\exp[i\xi_2(N-1)] \tag{18}$$

in the layers with $n=n_2$.

Let us discuss these expressions. First, it should be noted that all parameters for wave $E_2(z)$ can be expressed in terms of parameters for $E_1(z)$ by making the sign replacements $\xi_2=-\xi_1$, $\varphi_2=-\varphi_1$, $\psi_2=-\psi_1$. Second, all these parameters can be either real or complex. However, using formulas (11)–(13), we can establish a simple relation. If $\xi_{1,2}$ is complex, $\varphi_{1,2}$ and $\psi_{1,2}$ are real, and we can see from Eqs. (17) and (18) that the waves $E_1(z)$ and $E_2(z)$ lie in forbidden regions. If $\xi_{1,2}$ is real, $\varphi_{1,2}$ and $\psi_{1,2}$ are complex, and the waves $E_1(z)$ and $E_2(z)$ lie in allowed regions. Thus, we can determine the widths of the forbidden and allowed regions not only from the dispersion equation (13), but also from expressions (11) and (12) for $\varphi_{1,2}$ or $\psi_{1,2}$.

III. REFLECTION AND TRANSMISSION COEFFICIENTS FOR THE CASE OF NORMAL INCIDENCE

To illustrate the advantage of the present method, we consider the case of electromagnetic waves normally incident upon a two-layered periodic structure. In particular, we derive the reflection and transmission coefficients for the structure in an exact analytic form that is very convenient for subsequent analysis.

Denote the index of refraction of the medium on either side of the structure as n_0, i.e., $n=n_0$ if $z<0$, and $n=n_0$, if $z>Nd$. Assume that an electromagnetic wave with wave vector in vacuum k_0 is normally incident upon the two-layer periodical structure from the region $z<0$. In this case of normal incidence, light polarization does not play a role and the calculations given below are valid for both possible cases of polarization (along the y or x axis). Taking the amplitude of the incident wave as unity, we can express the total electric field in the region $z<0$ as

$$E(z)=\exp(ik_0n_0z)+A\exp(-ik_0n_0z), \tag{19}$$

where $A\exp(-ik_0n_0z)$ is the reflected wave, and A is its amplitude. In the region $z>Nd$ the total field is just the transmitted wave, which can be written in the form

$$E(z)=B\exp[ik_0n_0(z-Nd)], \tag{20}$$

where B is the amplitude. Using continuity conditions (function itself plus its derivative) for the total field $E(z)=C_1E_1(z)+C_2E_2(z)$ inside the structure, where $E_1(z)$ and $E_2(z)$ are defined by expressions (17), (18), with the field (19) at the point $z=0$ and with the field (20) at the point $z=Nd$ we obtain a system with four equations and four unknown coefficients C_1, C_2, A, B. Solving it for the coefficients A and B and taking into account the relations $\xi_1\equiv\xi=-\xi_2$ and $\varphi_2=-\varphi_1$, we have the final result

$$A=\frac{(1-b)\sin\Omega+a(b+1)\sin\Delta-2a\sqrt{b/(1-a^2)}(\cos\Omega-\cos\Delta)i}{-[(1+b)\sin\Omega+a(1-b)\sin\Delta]\pm2\sqrt{b(1-a^2)}\sin\xi\cot(N\xi)i}, \tag{21}$$

$$B=\frac{\pm2\sqrt{b(1-a^2)}[\sin\xi/\sin(N\xi)]\,i}{-[(1+b)\sin\Omega+a(1-b)\sin\Delta]\pm2\sqrt{b(1-a^2)}\sin\xi\cot(N\xi)i}, \tag{22}$$

where $b=n_0^2/(n_1n_2)$, a, Ω, Δ are determined by Eq. (10), and for our case of propagation along the z axis, for which $\theta=0$ and as a result $\theta_1d_1=k_0n_1d_1$, $\theta_2d_2=k_0n_2d_2$, take the simpler forms

$$a=\frac{n_2-n_1}{n_2+n_1}=\left(1-\frac{n_1}{n_2}\right)\bigg/\left(1+\frac{n_1}{n_2}\right),$$

$$\Omega=k_0(n_2d_2+n_1d_1),\quad \Delta=k_0(n_2d_2-n_1d_1), \tag{23}$$

and ξ ($\xi\equiv\xi_1$) is determined by Eq. (16). The upper sign in formulas (21), (22) corresponds to the case where $a\sin[\frac{1}{2}(\Omega+\Delta)]\geqslant0$ and the lower sign corresponds to the case where $a\sin[\frac{1}{2}(\Omega+\Delta)]<0$.

We can see that the reflection and transmission coefficients depend on five parameters: a, b, Ω, Δ, N (ξ is a function of a, Ω, and Δ). Each of them has clear physical meaning. a characterizes the variation in optical properties from one layer to the next (i.e., the amount of optical ''modulation''). It depends only on the ratio n_1/n_2, and for common dielectrics (dielectrics with positive ε) $-1<a<1$. b characterizes the Fresnel interaction between an electromagnetic wave and the boundaries of the structure, $\Omega=k_0(n_1d_1+n_2d_2)d/d=k_0nd$ is a period-average dimensionless wave vector of the light inside the structure, $\Delta=k_0(n_2d_2-n_1d_1)$ is the difference in optical paths of the wave inside each layer, and N is the number of periods.

Taking into account the fact that the term $\sin\xi\cot(N\xi)$ is always real, we can express the reflection and transmission coefficients as

$$R=\frac{[(1-b)\sin\Omega+a(b+1)\sin\Delta]^2+[4a^2b/(1-a^2)](\cos\Omega-\cos\Delta)^2}{[(1+b)\sin\Omega+a(1-b)\sin\Delta]^2+4b(1-a^2)[\sin\xi\cot(N\xi)]^2}, \tag{24}$$

$$T=\frac{4b(1-a^2)[\sin\xi\sin^{-1}(N\xi)]^2}{[(1+b)\sin\Omega+a(1-b)\sin\Delta]^2+4b(1-a^2)[\sin\xi\cot(N\xi)]^2}. \tag{25}$$

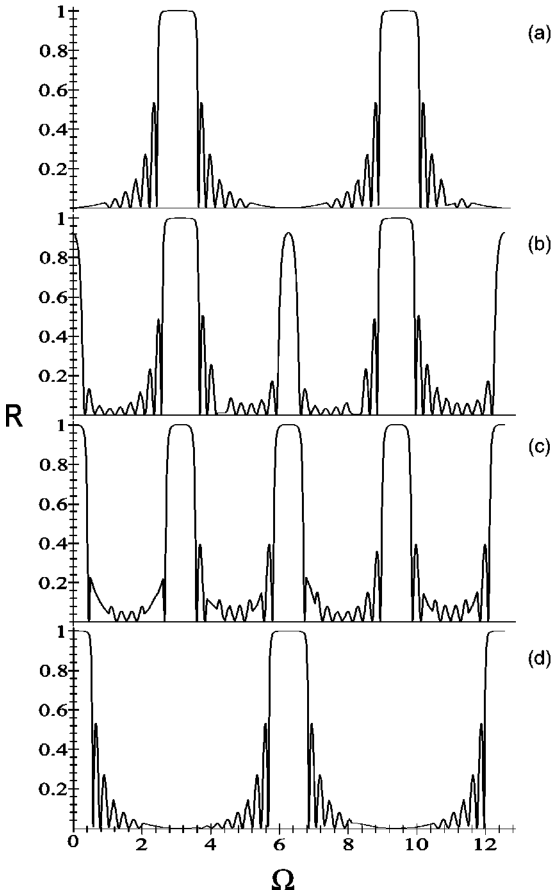

FIG. 2. Dependence of $R(\Omega)$ on Δ, for $n_0=2$, $n_1=1.5$, $n_2=2.5$, $N=10$. (a) $\Delta=0$, (b) $\Delta=\pi/4$, (c) $\Delta=\pi/2$, (d) $\Delta=\pi$.

It is evident that the reflection and transmission coefficients do not depend on the sign of ξ. Thus the dispersion equation (13) can be used to find ξ if the wave amplitudes themselves are not needed. Note that all our final results are expressed in terms of Ω and Δ, rather than $k_0n_2d_2$ and $k_0n_1d_1$ ($2k_0n_1d_1=\Omega-\Delta$, $2k_0n_2d_2=\Omega+\Delta$). We consider that this notation makes more physical sense. It facilitates comparisons between the exact method used here, and approximate methods [7,12] (Floquet-Bloch formalism, coupled wave theory) in which Ω is considered as the main variable and also allows one to consider some interesting limiting cases of our structure (see Appendix).

Now we investigate in detail how the general expression (24) for the reflection coefficient depends on the parameters Ω, Δ, a, b, N. For convenience, we take Ω as an independent variable and study the function $R=R(\Omega)$ with Δ, a, b, N as parameters.

First, we consider the function $R=R(\Omega)$ for different values of Δ. Suppose that $a=0.25$, $b=16/15$, and $N=10$ (for example, $n_0=2$, $n_1=1.5$, $n_2=2.5$). Figure 2(a) shows a plot of the graph of $R(\Omega)$ with $\Delta=0$, i.e., when the basic layers have equal optical thickness. As can be seen, the behavior of the curve is quite different in the forbidden ($|\cos\xi|>1$) and allowed ($|\cos\xi|>1$) regions. In the forbidden regions, R is almost constant and for given a and b it very nearly reaches unity. In the allowed regions, its dependence on Ω is oscillatory with increasing amplitude near the boundaries with the forbidden regions. With $\Delta=0$ there are only *odd* forbidden regions. Figure 2(b) shows the effect of introducing a difference in optical thickness of the basic layers. Then *even* forbidden regions appear. With $\Delta=\pi/2$, the widths of *even* and *odd* forbidden regions become equal [Fig. 2(c)]. With $\Delta=\pi$ [Fig. 2(d)], the *odd* forbidden regions disappear, leaving only *even* forbidden regions, which now have their maximum widths. If Δ is further increased, the pattern proceeds in the opposite direction so that when $\Delta=2\pi$ we have returned to the initial picture corresponding to $\Delta=0$. The cycle then repeats with period $\Delta=2\pi$.

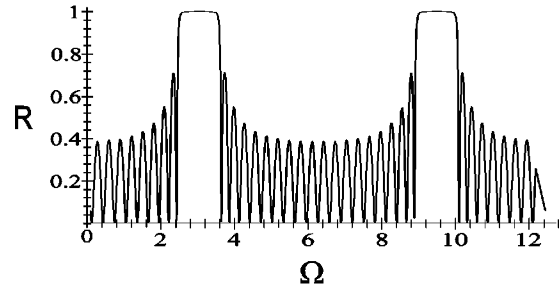

FIG. 3. Dependence of $R(\Omega)$ for $n_0=4$, $n_1=1.5$, $n_2=2.5$, $N=10$, $\Delta=0$.

Now we consider the influence on R of Fresnel interaction at the boundaries of the structure. For this we fix $\Delta=0$ and increase n_0 until $n_0=4$, corresponding to $b=64/15$ (Fig. 3). We can see [compare with Fig. 2(a)] that increasing b changes the behavior of R in the allowed regions, making the amplitude of oscillations higher. However, the behavior of R in the forbidden regions does not change.

In order to follow the influence on R of the optical modulation parameter a and the number of periods N, we minimize Fresnel interaction at the boundaries by taking $b=1$, i.e., $n_0^2=n_1n_2$, and take n_1 and n_2 to be 1.5 and 1.6, respectively, while keeping $N=10$ [Fig. 4(a)]. We can see that decreasing a causes the widths of the allowed regions to increase, with R tending to zero. R in the forbidden regions is also diminished, but it remains relatively high compared with R in the allowed regions. However, if we increase the number of periods N, for example, up to $N=50$, keeping all other

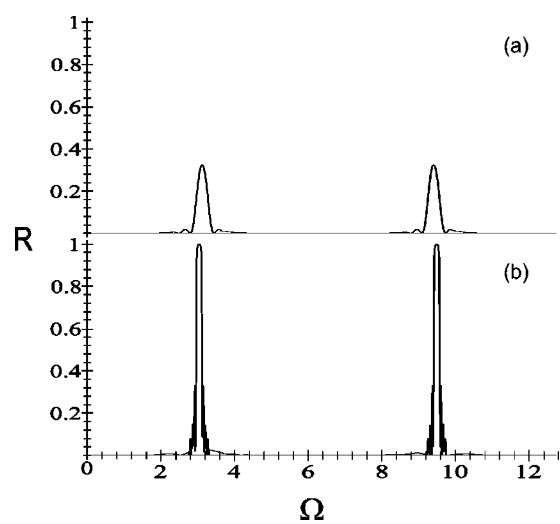

FIG. 4. Dependence of $R(\Omega)$ on N; for $n_0^2=n_1n_2$, $n_1=1.5$, $n_2=2.5$, $\Delta=0$. (a) $N=10$, (b) $N=50$.

parameters the same, we can see [Fig. 4(b)] that R in the forbidden regions almost reaches unity again. Thus, the most important and physically interesting influence of the number of periods on R is the following: even though the optical modulation a may be very small, there are always some narrow forbidden regions in which R practically reaches unity if the number of periods is sufficiently great.

IV. ELASTIC STRESS INFLUENCE ON THE REFLECTION AND TRANSMISSION

We consider the problem of an elastic stress influence on the reflection and transmission of the electromagnetic wave propagating in a two-layered periodic dielectric structure in order to demonstrate its possible application for the creation of a new optical switching system. Different types of such systems are widely used in various optical devices to control and govern laser radiation, as recently reviewed in Refs. [21,22]. The main reason for suggesting optical switch is to change the structure parameters so as to shift Ω from a forbidden region of optical frequencies, where the reflection coefficient almost reaches unity, to an allowed region, where the reflection coefficient can be around zero.

As we have in the preceding section, the parameters of the two-layered periodic dielectric structure which define the propagation of the electromagnetic wave with fixed wavelength are the widths of the basic layers d_1 and d_2 and their refraction indexes n_1 and n_2. Suppose we change each of them by a small amount $\delta d_1,\ \delta d_2,\ \delta n_1,\ \delta n_2$ keeping the periodicity. Then the variations in Ω and Δ have the form

$$\begin{aligned}\delta\Omega&=k_0(n_2\delta d_2+n_1\delta d_1+\delta n_2 d_2+\delta n_1 d_1),\\ \delta\Delta&=k_0(n_2\delta d_2-n_1\delta d_1+\delta n_2 d_2-\delta n_1 d_1)\end{aligned}\tag{26}$$

under the condition that we neglect second-order terms $\delta n_1\delta d_1$ and $\delta n_2\delta d_2$. There are also changes in a and b, but they are, respectively, not as big as $\delta\Omega$ and $\delta\Delta$. As a result, for a fixed wavelength λ_0 ($k_0=2\pi/\lambda_0$), the reflection and transmission coefficients (24), (25) are also changed. From the analysis in Sec. III it follows that in some cases the difference between the $R(\Omega_i,\Delta_i)$ and $R(\Omega_f,\Delta_f)$ where $\Omega_f=\Omega_i+\delta\Omega$, $\Delta_f=\Delta_i+\delta\Delta$ can be as much as 70–90 %. In order to get such a difference we should have Ω_i (the value of Ω before variations of the structure parameters) in the forbidden region and Ω_f (the value of Ω after variations) in the allowed region.

For the real creation of the variation of the structure parameters we can use variations of the elastic stress inside the structure. In order to keep the existing periodicity, we have two possibilities. First, we can modulate the structure by an acoustic wave satisfying the conditions $\frac{1}{2}\lambda_1=d_1$, $\frac{1}{2}\lambda_2=d_2$, where $\lambda_{1,2}$ are acoustic wavelengths in the basic layers $n_{1,2}$. Second, we can apply constant compression forces to the boundaries of our structure. In both cases the variations of the structure parameters can be described as

$$\delta d_{1,2}=S_{1,2}d_{1,2},\quad \delta n_{1,2}=-\frac{1}{2}{n_{1,2}}^3p_{1,2}S_{1,2},\tag{27}$$

where $S_{1,2}$ are the stresses in the layers $n_{1,2}$, $p_{1,2}$ are the elasto-optic coefficients in these layers, which depend on the material and directions of the stress and electromagnetic wave propagation.

From a mathematical point of view, the last case is simpler for analytical description because the $S_{1,2}$ are constant inside the basic layers. Let us consider this case in detail. Suppose we apply compression forces to the boundaries so that stress appears only along the z axis (axis of the periodicity of the structure). Then the variations of the structure parameters are

$$\begin{aligned}&\delta d_1=\frac{T_3}{c_{11}^{(1)}}d_1,\quad \delta d_2=\frac{T_3}{c_{11}^{(2)}}d_2,\\ &\delta n_1=-\frac{1}{2}n_1^3p_1\frac{T_3}{c_{11}^{(1)}},\quad \delta n_2=-\frac{1}{2}n_2^3p_2\frac{T_3}{c_{11}^{(2)}},\end{aligned}\tag{28}$$

where $c_{11}^{(1,2)}$ are stress coefficients of the layers with $n_{1,2}$, $p_{1,2}$ are their elasto-optic coefficients, and T_3 is a traction force (force per unit area) along the z axis. According to Eq. (26) the changes in Ω and Δ are then

$$\begin{aligned}\delta\Omega&=k_0\left[n_2d_2\frac{T_3}{c_{11}^{(2)}}\left(1-\frac{n_2^2}{2}p_2\right)+n_1d_1\frac{T_3}{c_{11}^{(1)}}\left(1-\frac{n_1^2}{2}p_1\right)\right],\\ \delta\Delta&=k_0\left[n_2d_2\frac{T_3}{c_{11}^{(2)}}\left(1-\frac{n_2^2}{2}p_2\right)-n_1d_1\frac{T_3}{c_{11}^{(1)}}\left(1-\frac{n_1^2}{2}p_1\right)\right].\end{aligned}\tag{29}$$

Let us demonstrate that even small variations (less than 0.1%) of the structure parameters may cause a large change in the reflection coefficient (24). As a first example, consider a two-layered periodic structure consisting of $N=20$ periods of polystyrene ($n_1=1.59$, $c_{11}^{(1)}=0.58\times10^4$ N/mm^2, $p_1=0.31$ [12,23]) with $d_1=7.5$ μm and chlorotellurite glass ($n_2=2.00$, $c_{11}^{(2)}=4.25\times10^4$ N/mm^2, $p_2=0.09$ [24]) with $d_2=5.9$ μm. Outside the structure there is also polystyrene ($n_0=1.59$). Figure 5(a) represents the reflection coefficient dependence on λ_0. We can see that for $\lambda_0=0.6328$ μm (He-Ne laser) the reflection coefficient almost equals 100% (no transmission).

Now suppose we apply constant compression forces to the boundaries of the structure so as to create a stress along the z axis (axis of the periodicity of the structure and the direction of the electromagnetic wave propagation). Let $T_3=-30$ N/mm^2, then $\delta d_1=-0.039$ μm, $\delta d_2=-0.004$ μm, $\delta n_1=0.0032$, $\delta n_2=0.0003$ and the reflection coefficient is plotted on Fig. 5(b). We can see now that for $\lambda_0=0.6328$ μm R is less than 10%, i.e., the reflection is decreased by a factor of 10 (in fact we have almost full transmission). Figure 6 illustrates general dependence of the reflection coefficient R on the applied traction force T_3 for this structure.

As a second example we consider the structure with $N=40$ periods of fused silica ($n_1=1.457$, $c_{11}^{(1)}=7.85\times10^4$ N/mm^2, $p_1=0.27$ [25,23]) and flint glass ($n_2=1.616$, $c_{11}^{(2)}=4.61\times10^4$ N/mm^2, $p_2=0.256$ [25]) which is surrounded by the flint glass itself ($n_0=1.616$). Let $d_1=5.1$ μm and $d_2=4.6$ μm. Then the reflection coefficient

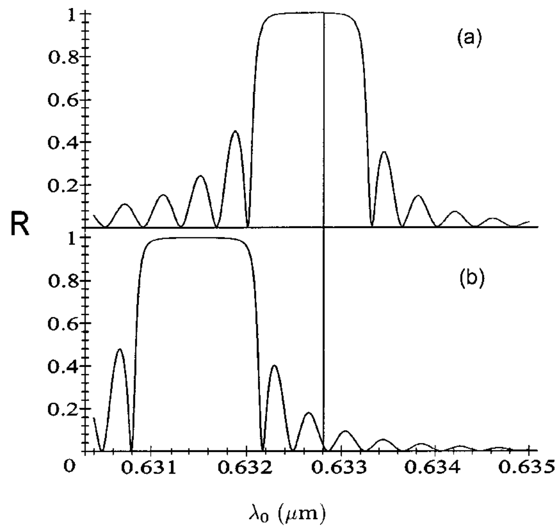

FIG. 5. Dependence of R on λ_0 for $n_0=1.59$, $N=20$. (a) $n_1=1.59$, $n_2=2.0$, $d_1=7.5$ μm, $d_2=5.9$ μm (no stress is applied), (b) $n_1=1.59+0.0032$, $n_2=2.00+0.0003$, $d_1=7.5-0.039$ μm, $d_2=5.9-0.004$ μm (stress is applied).

dependence on λ_0 is plotted in Fig. 7(a). We can see again that $R(0.6328\ \mu\text{m})\approx 1$. If we apply traction force $T_3=-70$ N/mm^2 to the boundary of the structure along the z axis and corresponding forces along the x and y axis (in order to have stress just along the z axis) the variations of the structural parameters will be $\delta d_1=-0.005$ μm, $\delta d_2=-0.007$ μm, $\delta n_1=0.0004$, $\delta n_2=0.0008$, and the reflection coefficient $R\ (0.6328\ \mu\text{m})\approx 0$ [Fig. 7(b)], i.e., the situation is exactly the same as in the previous example. The application of stress changes the characteristics from no transmission to nearly full transmission. Figure 8 illustrates general dependence of the reflection coefficient R on the applied traction force T_3 for this structure.

From these examples we can see general requirements for using a two-layered periodic structure as a basic medium for optical switches. The most important one is to build the structure from materials with very high elasticity such as, for example, polymers.

A practical difficulty in achieving this goal will be the sensitivity of the transmission and reflection coefficients to fluctuations in thickness of the layers due to inhomogeneous growing conditions. This will tend to smear out the sharp features shown in Figs. 5–8. In order to study this effect mathematically, we are currently developing a Green function technique for layered periodic structures. Our aim is to obtain an exact analytic form which will allow us to study in a precise way the effect of fluctuations in thickness on the reflection and transmission coefficients. The results will perhaps help to identify regions of stability where the coefficients are not critically sensitive to fluctuations in the structure parameters. We plan to present the results in the future.

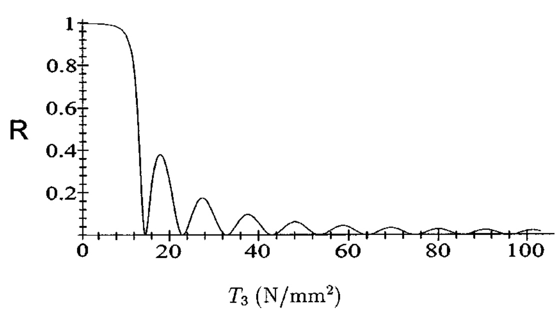

FIG. 6. Dependence of R on T_3 for $n_0=1.59$, $N=20$ with unstressed parameters $n_1=1.59$, $n_2=2.0$, $d_1=7.5$ μm, $d_2=5.9$ μm.

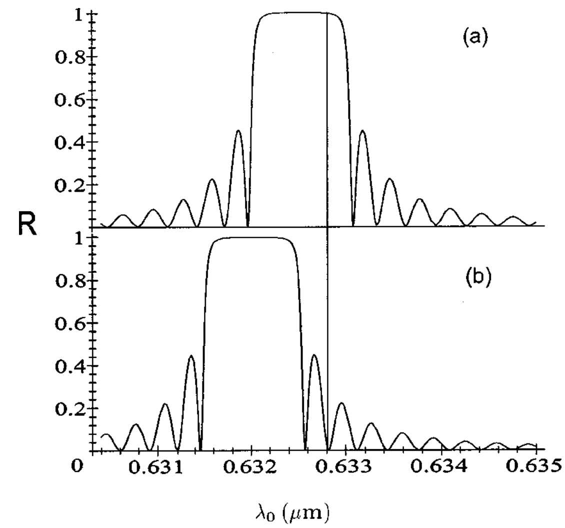

FIG. 7. Dependence of R on λ_0 for $n_0=1.616$, $N=40$. (a) $n_1=1.457$, $n_2=1.616$, $d_1=5.1$ μm, $d_2=4.6$ μm (no stress is applied), (b) $n_1=1.457+0.0004$, $n_2=1.616+0.0008$, $d_1=5.1-0.005$ μm, $d_2=4.6-0.007$ μm (stress is applied).

V. CONCLUSION

We have applied Floquet-Bloch theory to the well-known problem of the propagation of electromagnetic waves through a two-layered periodic dielectric structure, using an exact analytical method. The main idea of this method is to represent the solution for each Floquet-Bloch wave inside each basic layer as a sinusoidal function. This allows us to determine general expressions for the dependence of the reflection and transmission coefficients on the structure parameters in a more physically transparent form. Using the results of this analysis we found that small variations of the structure parameters can lead to large changes in the reflection and transmission coefficients. In particular, we demonstrated the possibility of changing the reflection and transmission of

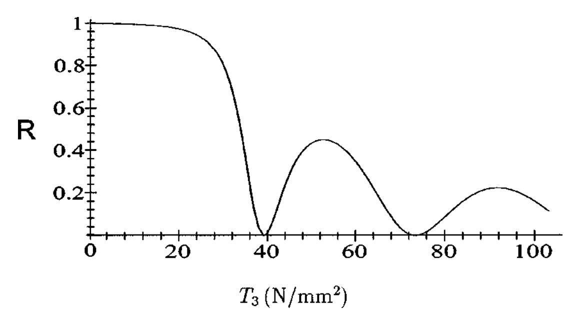

FIG. 8. Dependence of R on T_3 for $n_0=1.616$, $N=40$ with unstressed parameters $n_1=1.457$, $n_2=1.616$, $d_1=5.1$ μm, $d_2=4.6$ μm.

the electromagnetic wave propagating in a two-layered periodic structure by up to 90% by the application of a constant elastic stress inside the structure. By a judicious choice of materials we have found two cases where a practical optical switch based on these properties would be feasible.

APPENDIX

In order to obtain a better physical understanding of expressions (24) and (25) for the reflection and transmission coefficients we consider some special and limiting cases of our structure.

(a) Example 1. Let $n_1=n_2\equiv n$. Then $a=0$ and $b=n_0^2/n^2$. Formula (16) gives $\xi_1\equiv\xi=-\Omega$, and expression (21) for the amplitude reflection coefficient takes the form

$$A=\frac{(1-n_0^2/n^2)}{-(1+n_0^2/n^2)-2(n_0/n)\cot(N\Omega)i}. \tag{A1}$$

This is the reflection coefficient from one layer with refractive index n and the width Nd $(\Omega=k_0nd)$.

(b) Example 2. Suppose that one basic layer, for example, the layer with $n=n_2$, is a half-wave layer, i.e., $k_0n_2d_2=l\pi$, where $l=1,2,3,\ldots$. Then, $\xi_1=-k_0n_1d_1$ and

$$A=\frac{(1-n_0^2/n_1^2)}{-(1+n_0^2/n_1^2)-2(n_0/n_1)\cot(Nk_0n_1d_1)i}. \tag{A2}$$

This is the reflection coefficient from a single layer with refractive index n_1 and width Nd_1. The result is in accordance with the well-known fact that any numbers of half-wave layers do not influence reflection or transmission. In the case where both basic layers are half-wave layers, $A=0$ and $B=1$, i.e., we have full transmission.

(c) Example 3. Suppose that the basic layers are quarter-wave layers, i.e., $k_0n_1d_1=k_0n_2d_2=\pi/2$. Then $\Omega=\pi$, $\Delta=0$, and Eq. (21) immediately gives the reflection coefficient

$$A=\frac{1-(n_1/n_2)^{2N}}{1+(n_1/n_2)^{2N}}, \tag{A3}$$

in agreement with Born and Wolf [9].

(d) Example 4. Let us consider the case where $b=1$, i.e., $n_0^2=n_1n_2$. Physically, this case means that general Fresnel interaction at the boundaries of the structure has a minimum. Then Eq. (24) takes the form

$$R=a^2\frac{\sin^2\Delta+[1/(1-a^2)](\cos\Omega-\cos\Delta)^2}{\sin^2\Omega+(1-a^2)[\sin\xi\cot(N\xi)]^2}. \tag{A4}$$

This result illustrates the interesting property that $R(n_1/n_2)=R(n_2/n_1)$. Thus the intensity of the reflected wave does not change under an interchange of the basic layers.

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